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Speaking of Graphics

Chapter 3

Oresme, Bacon, Descartes and Coordinate Geometry

3.1. Nicole Oresme and the Geometrization of Physics

3.1.1. Life and work of Oresme

The medieval figure of Nicole Oresme is of great importance in the history of the graphical representation of quantitative data. He rightly may be considered as the one who prepared the field for the modern ideas that were developed subsequently during the Renaissance and the Enlightenment and which still permeate our current thinking and practices. Biographical details on Oresme are rather scarce. He lived from 1320 to 1382. He studied Artes at the University of Paris under the famous nominalist philosopher Jean Buridan [13]. He was nominated bishop of Lisieux in Normandy in 1377.
Oresme lived in a period in which the foundations for the new physics of the 16th century were laid. This evolution took place almost simultaneously and independently in three different schools of thought. Jean Buridan headed the school of French nominalists in Paris. His most renowned students were Nicole Oresme, Albertus of Sachsen and Marsilius of Inghen. Thomas Brandwardine had formed a school of nominalist philosophers at Merton College in Oxford, which included the famous Richard Swineshead (or Suisset) also known as Calculator and whose followers were referred to as Calculatores. Finally, in Italy, there was the school of Averroists [14]. These three schools had in common that they tried to overcome the prevailing Aristotelian philosophy with respect to natural phenomena [5].

Oresme is known as a popularizer of science. He translated Latin works into French [1]. He is also credited to have elaborated the first monetary theory. Oresme contributed to the mathematical knowledge of his time, e.g. by his work on infinite series [7]. His main interest, however, was in the study of the structure of matter and in kinematics. He defended the physical theory of the elements (earth, fire, air and water) against the Aristotelian-scholastic theory of form and substance [3]. Form is an active principle of being which manifests itself as actuality ('energeia'), while substance is a passive principle which is present in the form of potentiality ('dynamis'). According to Aristotle, life is a continuous transition between potentiality and actuality. Oresme also defended the impetus theory against the prevailing Aristotelian theory of the intelligences. The impetus theory maintained that movement continues forever in the absence of a force, while the intelligences theory proposed that sustained movement requires the persisting action of a force [2].

Oresme is best known, however, for his introduction of the concept of 'form latitudes' and their various configurations. In order to reproduce some of the flavour of the epoch, we will maintain some of the original Latin terminology in the text and illustrations.

3.1.2. Form latitudes

If we consider the cross-section of an object ('subjectum'), then the extension of this cross-section can be chacterised geometrically by a magnitude which Oresme called longitude ('linea extensionis'). (For convenience, we assume here that the object is thin and flat.) In addition to extensive properties, Oresme also considered intensive properties, such as temperature, density, velocity, etc. which could vary from one point to another. (In Aristotelian scholastic terminology, these properties defined the variable 'form' of an object, as opposed to its immutable 'substance'.) The invention of Oresme was to represent the magnitude of the property (or form) by means of line segments ('lineae intensionis') which were drawn perpendicularly to the direction of the longitude. These line segments were referred to as form latitudes or latitudes for short (Fig. 3.1.1a). The extremities of the latitudes were also thought to build a continuous curve ('linea summitatis') which is characteristic for the object at the given longitudinal cross-section (Fig. 3.1.1b). Particular attention was given to the area under the curve ('figura qualitatis') which was thought to be representative for the particular quality of the object (temperature, density, velocity, etc.). This area was referred to as the configuration of the latitudes, and was the particular subject of investigation of Oresme [4].

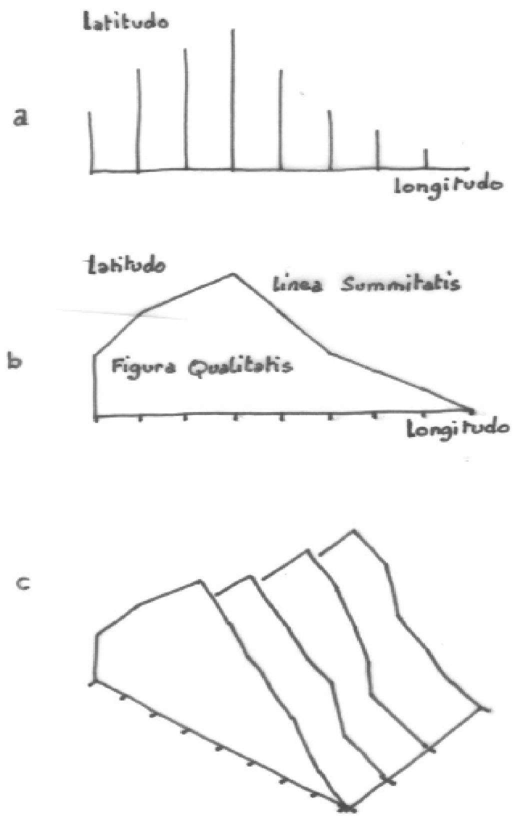


Figure 3.1.1. Diagrams of form latitudes according to Nicole Oresme. (**a**) The case of a one-dimensional cross-section which defines the longitude. The intensity of a form (i.e. a quality such as heat) at each point of the longitude is defined by the latitude which is oriented perpendicularly to the longitude. (**b**) The endpoints of the form latitudes build a continuous curve ('linea summitatis') which defines the characteristic configuration of the form. The area under the curve represents a quantitative measure for the quality represented by the latitudes ('figura qualitatis'). (**c**) Parallel cross-sections define a superficial configuration of form latitudes. A full three-dimensional object is thought of as built up from parallel two-dimensional cross-sections, each endowed with its own superficial configuration[12].

The concept of form latitudes was extended to a series of parallel one-dimensional cross-sections of the object. In this case the extremities of the latitudes build a surface (Fig. 3.1.1c). It was also extended to three-dimensional objects by considering parallel slices of the object, each generating a particular surface of form latitudes. (A high-dimensional space of form latitudes was not considered at that time.)

The importance of the form latitudes lies in the fact that, for the first time, physical quantities, other than geometrical or geographical dimensions, are represented graphically by means of coordinates. It has been claimed by Pierre Duhem that Oresme had preceded Descartes' discovery of coordinate geometry by almost three hundred years [6], but this conclusion seems to be too strong. One can state, however, that the seeds of the new ideas of the 16th and 17th century have been laid during the scholastic period by natural philosophers who challenged the dominant Aristotelian and metaphysical concepts. This process was initiated in the 13th century by Albertus Magnus and Roger Bacon and was continued thereafter by William of Ockham and Nicole Oresme [15].

Oresme's original contribution can best be described as a first attempt at the geometrization of physical properties of matter. According to Oresme, the perception of a physical quantity (such as the temperature of an object) was attributed to the particular configuration of the surface of form latitudes. Different properties of an object were thus explained in terms of different configurations of the corresponding physical qualities. This was not an unusual thought in medieval times, as it fell in line with the ancient atomistic theory of Democritus, which maintained that the properties of an object were due to the particular shape of its constituent atoms (smooth, peaked, etc.) [12]. In medieval thinking these configurations were not merely a convenient graphical representation of physical reality, they were the reality of the physical phenomenon itself. Thus, an object, in addition to its (tangible) extensive dimensions or longitudes, was also thought to possess (invisible) intensive dimensions or latitudes. From this point of view, the configurations were more than a graphical illustration of physical concepts and mathematical calculations. In the late Middle Ages, they contributed to a better understanding of the concepts of velocity and acceleration [8].

Oresme also applied his concept of form latitudes to the study of music. Here he made a distinction between the loudness of a tone ('fortitudo', an extensive quantity) and its frequency ('acuties', an intensive quality). The loudness together with the accompanying configuration of frequencies was thought to define the aesthetic qualities of music. One may regard this idea of dividing a phenomenon into an extensive quantity and intensive qualities as a forerunner of the distinction between size and shape, to which we will return in a later chapter on multivariate analysis [3].

The influence of Oresme in his own time was rather small, except for a few German scholars (Albertus of Sachsen and Marsilius of Inghen) who studied with him in Paris and some Italian followers [3]. He also received little recognition from the

Mertonian school of nominalist philosophers at Oxford. According to Duhem, however, his ideas were known to Galilei and to Descartes.

3.1.3. Functions

Thomas Brandwardine had formulated a law which stated that the velocity of an object is directly proportional to the force that acted upon it and inversely proportional to the resistance that it met with [9]. (One should be aware that the concept of force in the fourteenth century was not as sharply defined as today.) This so-called rule of proportions was generally accepted by the followers of Richard Swineshead (Calculator) [10]. In Oresme's time the rule of proportions for the velocity of an object was not yet understood as an algebraic function. It was part of the literal algebra which used verbal descriptions rather than algebraic expressions. Nor were there clear ideas about the different types of velocity, such as occur in uniform motion (at constant velocity) and in uniformly accelerated motion (at velocity which increases at a constant rate).

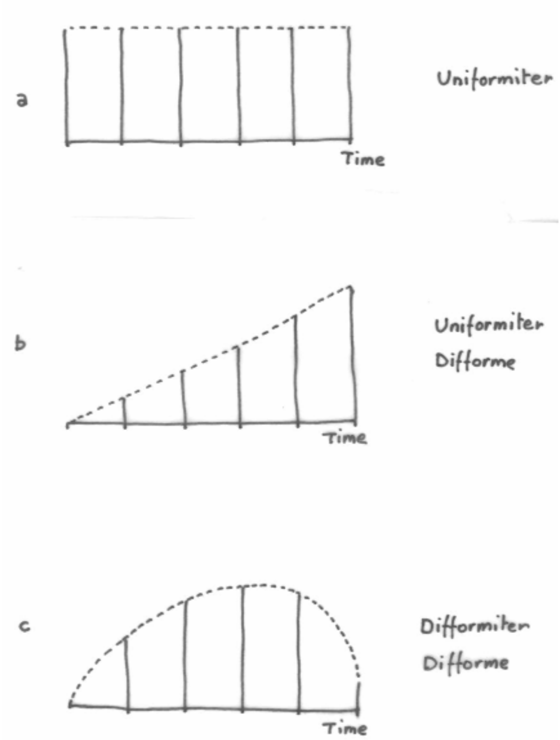


Figure 3.1.2. Diagrams of form latitudes in which the longitude represents different intervals of time. The latitude may represent a form (or quality) which varies with time (e.g. a quality such as velocity). The configuration of form latitudes applies to a particular point of the object. (**a**) Rectangular ('uniformiter') configuration, e.g. of an object moving at constant speed. (**b**) Triangular ('uniformiter difforme') configuration, e.g. of an object which is subjected to a constant acceleration. (**c**) Curvilinear ('difformiter difforme') configuration, e.g. of a ballistic projectile [12].

The configurations of Oresme helped to clarify cinematic concepts. To this effect, he considered the time during which an object is observed as the extensive dimension or longitude. In this case, the instantaneous velocity of the object forms is the intensive quality or latitude, which is represented perpendicularly to the longitude (Fig. 3.1.2). The extension of geometrical dimensions of the object ('quoad subjectum') with a time dimension ('quoad tempus') was a formidable leap of imagination. As a result, the configurations of Oresme allowed symbolizing the different types of motion in a graphical way and this prepared the ground for the elaboration of algebraic functional relationships. Oresme, however, did not take the step towards the formulation of functional relationships. Instead he focused his attention on the graphical configuration, i.e. the area ('figura qualitatis') that is formed under the curve ('linea summitatis') that joins the end points of the latitudes ('lineae intensionis'). In the case of a moving body, he distinguished between rectangular ('uniformiter'), triangular ('uniformiter difforme') and semicircular ('difformiter difforme') movements (Figs. 3.1.2a, b and c). An excerpt from the printed edition of the 'Tractatus de Latitudinibus Formarum' is reproduced in Fig. 3.1.3 [11].

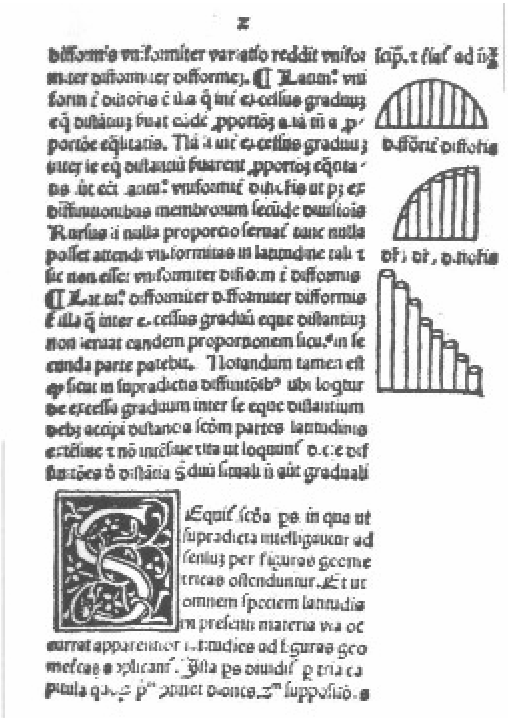


Figure 3.1.3. Illustration from the printed edition of Oresme's 'Tractatus de Latitudinibus Formarum' (1486), showing various diagrams of form latitudes. The illustration has been reproduced previously by Funkhouser [11].

In Oresme's thinking, a body is endowed during its movement with a particular qualitative form (in German 'Gestalt'), which is shown by the configuration of its velocities. Hence, a body moving at constant velocity has a form (Gestalt) which is generally different from that of a body which is accelerated constantly. In the former case it is rectangular; in the latter it is triangular. Since the area under the curve of latitudes is a measurable quantity, Oresme concluded that he had found a quantitative measure for qualities ('quantitas qualitatis') [5]. This allowed him to produce quantitative statements from the configuration of the velocity of a body. He came to the, for that time, astonishing finding that the effect of movement of two bodies is equivalent when the areas under the curve of their velocity configurations are the same. In particular, he discovered that the effect of motion of a constantly accelerated body is the same as that of a body moving at a constant velocity, which is determined at the midpoint of the configuration of the former (Fig. 3.1.4). In other terms, a triangular configuration is equivalent to a rectangular one, which possesses the same area. This property has been referred to by Duhem as Oresme's rule [5] and is still known as such. Anneliese Maier, however, has pointed out that the rule was already common knowledge in Oresme's time and was generally accepted, notably by the Calculatores at Oxford [5]. According to Dijksterhuis, this does not depreciate, however, the merit of Oresme for having seen the broader implications of the rule and for emphasizing its geometrical and graphical representation [12].

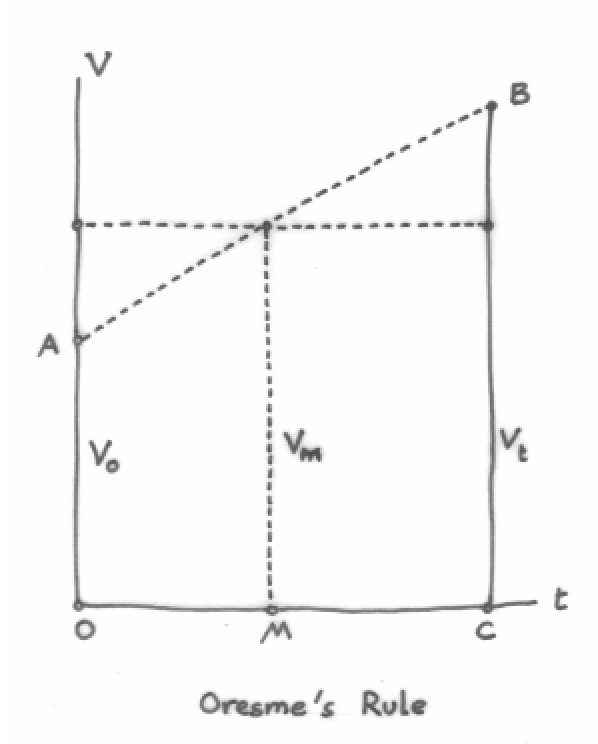


Figure 3.1.4. Oresme's rule states that the effect of a triangular configuration is equivalent to that of a rectangular one which assumes the intensity of the form (or quality) at the midpoint. In the case of a moving body, this rule has been interpreted as a forerunner of integration of the instantaneous velocity, which produces the distance travelled by the object [12].

It was realized by Oresme that the area under the curve ('quantitas velocitatis') represented the distance travelled by the object, to which he referred as 'velocitas totalis'. The latter can be translated freely into integral velocity. It is tempting, therefore, to conclude from the foregoing that the integral calculus had already been discovered in the late fourteenth century by Oresme and his contemporaries, three centuries before its formal definition by Newton and Leibnitz. One should be prudent, however, with statements of this sort, as the full implications of the new ideas were hardly realized by medieval natural philosophers. The concept of instantaneous velocity as the limit of the proportion of an infinitely small distance covered during an infinitely short lapse of time was unknown to them. They also lacked the methods and instruments to perform reliable observations which would have allowed the verification of their findings. In any case, there is no evidence that Oresme's rule has been applied in his own time to the study of a freely falling object under the constant acceleration of gravity. But, as already mentioned before, the intellectual climate was being prepared for the scientific revolution that was to come with Galilei, Kepler, Descartes and Newton [7, 12].

Our interest in Nicole Oresme derives specifically from his use of graphical representations as an instrument for the advancement of understanding of natural phenomena.

Notes to Oresme

[1] Nicole Oresme's works that survived include:
'Questiones de Caelo et Mundo', a treatise on matter, motion and force.
'Propositiones Proportionum' and 'Algorism Proportionum', on mathematical problems.
'Tractatus de Configurationibus Intensionum' (around 1350), otherwise known as the 'Tractatus de Uniformitate et Difformitate Intensionum', or as the 'Tractatus de Latitudinibus Formarum'. Printed in Padua, 1482 and 1486. This work contains Oresme's ideas on the geometrical representation of physical properties.
He also wrote about music, the monetary system and about some rather esoteric topics.

[2] Dictionary of scientific Biography. Ch. Scribner's Sons, New York, 1974.

[3] Jacob of Neapel and Blasius of Parma introduced the theory of form latitudes in Italy.
Anneliese Maier, An der Grenze von Scholastik und Naturwissenschaft. Edizione di Storia e Letteratura, Roma, 1952.
Anneliese Maier is one of the most authoritative sources of the period which covers the emergence of the new physics during the transition from the Middle Ages to the Renaissance.

[4] Anneliese Maier, Zwei Grundproblemen der scholastischen Philosophie. Das Problem der intensiven Groesse, die Impetustheorie. Edizioni di Storia e Letteratura, Roma, 1951.

[5] Anneliese Maier, Die Vorlaufer Galileis im 14. Jahrhundert. Studien zur Naturphilosophie der Spaetscholastik. Storia e Letteratura, Roma, 1949.

[6] Pierre Duhem, Etudes sur Leonard de Vinci, Ceux qu'il a lus et ceux qui l'ont lu. Serie I - III, Paris, 1906.

[7] Dirk J. Struik, A concise History of Mathematics. 1948. Dutch translation: De Geschiedenis van de Wiskunde. Spektrum, 1990, p. 113.
Nicole Oresme introduced a notation for the fractional powers of numbers. He also proved that the harmonic series is divergent.

[8] Ernst Borchert, Die Lehre von der Bewegung bei Niclaus Oresme. Beitraege zur Geschichte der Philosophie und Theologie des Mittelalters. Aschendorffscher Verlagsbuchhandlung, Muenster, 1934.

[9] Thomas Brandwardine, Tractatus Proportionum, 1328.
Brandwardine headed a school of nominalist philosophers (distinct from the followers of William of Ockham) at Merton College in Oxford. He was ordained archbishop of Canterbury in 1348.

[10] Richard Swineshead (Suisset), Liber Calculationum, 1350. He belonged to the school of Brandwardine at Oxford and was known as Calculator.

[11] H.G. Funkhouser, Historical development of the graphical representation of statistical data. Osiris, Vol.3, (Georges Sarton, ed.), St. Catherine Press, Bruges (Belgium), 1938, p.276.
The figures from Oresme's printed edition of the 'Tractatus de Latitudinibus Formarum' (which appeared in1486 in Padua) have been reproduced from the article of Funkhouser. Oresme described 13 forms of movement, each characterized by a particular configuration of velocities. The diagrams give the impression that the vertical latitudes are represented as organ pipes of different lengths. It is assumed that these diagrams are true copies of Oresme's original manuscripted notes, and that they have not been embellished afterwards by the printer.

[12] E.J. Dijksterhuis. The Mechanization of the Cosmos. Meulenhoff, Amsterdam, 1950., pp. 212-220. (In Dutch: De Mechanisering van het Wereldbeeld.)

[13] During the Middle Ages, philosophers were divided between nominalists and realists. The nominalist position held that abstract concepts or so-called 'universalia' (e.g. mankind, beauty, etc.) are only creations of the mind and have no existence beyond it. The realist position, on the contrary, affirmed that the 'universalia' exist either in a world beyond our senses (Plato) or within the observable world (Aristotle). In the 14th century, the nominalists became the leading opponents against the metaphysics of Aristotle and against the pedantic practices of scholastic philosophers. They played an important part in the transition toward the modern physical and mathematical sciences.

[14] Averroes (Ibn Roesjd, 1126-1198) was one of the most important Arabic commentators of Aristotle. He exerted great influence on Western philosophers during the Middle Ages. Averroes advocated the separation (dualism) between religious belief and rational thinking, which led eventually to the secularization of philosophy.

[15] Albertus Magnus (Doctor Universalis) (1200-1280) was the most distinguished theologist and natural philosopher of the middle ages. Thomas Aquinas studied under him. He tried to reconcile the Platonic and Aristotelian positions, by considering that the soul was both outside and within the material world. He also emphasized the importance of direct observation and experimentation.

Roger Bacon (Doctor Mirabilis) (+-1210-1292) made a distinction between knowledge obtained from speculative reasoning ('intellectus') and that brought to life by experience ('experimentum'). As a futurologist, he foresaw the invention of the aeroplane, submarine, automobile, etc.)

William of Ockham (Venerabilis Inceptor) (1285-1349) distinguished between knowledge derived from revelation, observation and logic. He proposed that, whenever possible, the simpler explanation should be preferred over the more complex according to his maxim 'pluralitas non est ponenda sine necessitate' (complexity should not be imposed unnecessarily), better known as Ockham's razor. Founder of a school of nominalist philosophers at Oxford which marked the transition between the 'via antiqua' of the scholastics and the 'via moderna' of those opposed to Aristotelian metaphysical doctrine.

3.2. Francis Bacon and latent forms in data.

3.2.1. Life and work of Bacon

Francis Bacon (Fig. 3.2.1) and René Descartes are the key figures that have laid the foundations of modern science during the first half of the 17th century. Although they were contemporaries, they worked independently and along different paths. The starting point of Bacon's philosophy of science was formed by observations and measurements from which he attempted to construct general laws, an approach which is referred to as inductionism. Descartes, on the other hand departed from a purely rational starting point, from which he derived consequences that could be tested empirically. This approach is called deductionism. Both have contributed, in their own way, to our present-day practice of graphical representation of quantitative data.

Figure 3.2.1. Francis Bacon, from the frontispiece of the 'Opera omnia' (1665).

To Francis Bacon we owe the systematic use of tabulated data, as an instrument of scientific enquiry. He also was the first to think in an organized way about science, which he conceived as a world-wide and concerted effort to obtain 'certain' knowledge from empirical observations. He also pointed out many pitfalls that may detract from this enterprise. Bacon's so-called naive inductionist approach has been corrected and improved in the centuries that followed, more recently by the logical positivists of the Wiener Kreis and by the critical inductionism of Popper [1]. Although it is no longer a tenable position in the philosophy of science, naive inductionism nevertheless merits to be discussed here, because of its historical relevance to the development of statistical graphics. We have based our historical account to a large extent on the biography by Anthony Quinton [2].

Francis Bacon was born in 1561 in London, as the son of Sir Nicholas Bacon, who was appointed to the distinguished post of Lord Keeper by Queen Elisabeth I. He entered Trinity College at Cambridge at the age of twelve, where he stayed for two years. At Cambridge, Bacon may have been under the influence of the teachings of the logician Ramus who opposed the philosophy of Aristotle [3]. After an official stay in France, he returned to London where he became a barrister and a member of the House of Commons. From then on, Bacon started a relentless and life-long campaign in order to seek royal favours and preferment. At first, under the reign of Elisabeth I, these efforts were rather unsuccessful. But, after the ascension of James I to the throne in 1603, his professional aspirations gradually became fulfilled. He was appointed successively to the posts of Solicitor-General, Attorney-General, Lord Keeper (the position held formerly by his father) and finally to Lord Chancellor, the highest legal post in the kingdom. In public life, Francis Bacon appeared as an unemotional, ambitious, cunning and, at some occasions, treacherous personality. The continuous struggle for high positions with his personal enemy and rival Edward Coke has become legendary. In 1621, however, at the height of his career misfortune struck when he had to admit to a charge of bribery. He was fined, sent to the Tower for two or three days, and, worst of all, banned from the royal court, which put an end to his legal and political activities.

Quite different is the picture of Francis Bacon that we gather from his literary, historical and legal work. To the student of literature Bacon is best known as the author of fifty-eight 'Essays', a collection of practical wisdom and aphorisms, which he compiled during the course of his life. He also made a rational analysis of myths and fables in the 'Wisdom of the Ancients', wrote a historical book on King Henry VII and contributed to the legal literature of his time in 'Maxims of the Law'.

Most extraordinary, however, is his philosophical enterprise which aimed at the foundation of science on the basis of empirical observation, organization of work and inductive reasoning. His revolutionary ideas were presented and illustrated in 'The Advancement of Learning' (1605), 'New Atlantis' (1610), 'The great Instauration' and 'Novum Organum' (1620). The title of the latter work was chosen such as to leave no doubt on Bacon's position against Aristotle whose major scientific work was entitled 'Organon' (meaning tool or device). The scientific renewal worked out in great detail by Francis Bacon is of interest to our study of the role of statistical graphics in discovery and hypothesis formation, although Bacon himself never made use of graphical means to illustrate his ideas.

One can only wonder how Bacon's legal, political, literary and scientific interests which could be united in one and the same person. There also remains the unresolved contrast between his conservative public appearance, which is often scorned at, and his revolutionary intellectual achievement, which is everlasting and monumental.

3.2.2. The three fallacious systems of learning

In 'The Advancement of Learning' and the 'Novum Organum' Bacon launched an attack on three prevailing systems of knowledge acquisition, to which he referred as delicate, disputatious and fantastic learning.

The ‘delicate learning’ was used by the humanists who found their inspiration in the ancient Greek and Latin authors. Bacon considered history (natural as well as civil), philosophy and poetry as major subdivision of his new science, which were thought to be amenable to experimental observation and inductive reasoning. In particular, he regarded history as the storeroom of certified knowledge, which was to be made available to the other branches of science.

The ‘disputatious learning’ was attributed to the scholastics, who were heavily criticized by Bacon for their sole reliance on logic of the mind rather than on observation by the senses, and their single-minded adherence to the teachings of Aristotle. Bacon stressed the unfruitfulness of syllogistic deductions from abstract and dogmatic principles that were not the result of empirical observation. A critique against Bacon was that he did hardly perform any measurements himself. He was also singularly ignorant of the mathematical and physical advances that had been made in his own time. His merit, however, has been to consider science as a grand collective undertaking in order to better understand the creation of God and for the material advancement of mankind. In his view, science was a cumulative process, with a rigorous subdivision of tasks, involving experimentation, compilation and interpretation of data, planning of new experiments, extraction of general laws, dissemination of discoveries, etc. In his 'New Atlantis', Bacon described a utopian state which was governed and organised for this purpose [4], much in the way as in Plato's Republic.

Finally, ‘the fantastic learning’ was denounced by Bacon. In this category he placed the esoteric, magic, folklore and alchemist traditions which were promoted in his time, notably by Fludd. His main objection was that these traditional sources were often accepted uncritically. Against these he placed the co-operative activity of organized and institutionalized science [5]. Bacon himself, however, has not been free from esoteric and mystical influences.

3.2.3. The four idols of the mind

In his 'Novum Organum' Bacon warned against four fallacies of the mind, which he gave the colourful names of the idols of the tribe, the cave, the market and the theatre. These idols are common prejudices of which we are often not aware and from which the mind should be purified if it is to reason free of error.

The’idol of the tribe ('idola tribus') symbolizes the bias that creeps into our judgement when we draw conclusions hastily, for want of obtaining a result. It is also present when we are carried away by an attractive hypothesis, which blinds our mind and senses for evidence to the contrary. This idol lends more weight than deserved to observations that corroborate a previously suggested general law.

The idol of the cave or den ('idola specus') refers to preferences that we have by virtue of nature and nurture. People gather stronger impressions of qualities for which they are particularly sensitive, or for whose perception they have been most intensively trained. The name of this idol was probably suggested by the story of the cave in the 'Republic' of Plato.

The idol of the market ('idola fori') addresses the confusion, uncertainty and nonsense that are often introduced by the use of incorrect, incomplete or inappropriate language.

Finally, the idol of the theatre ('idola theatri') accuses our reliance on authoritative sources. Bacon certainly took aim here at Aristotle and his scholastic adepts. In general, he denounced herewith the uncritical acceptance of statements deduced by logic from unproved assumptions. He also warned against the rash acceptance of empirical facts that have not been replicated and confirmed.

3.2.4. The three tables of discovery

In order to discover a 'first vintage of a law' from empirical observations, Bacon proposed the compilation of three types of tables, which are called respectively the table of presence (existence), the table of absence (deviation) and the table of degrees (comparative instances). The three tables of discovery are at the heart of Bacon's inductive method. They constitute the natural 'history' of the phenomenon that is under study. The ultimate purpose of the tables was to order facts in such a way that the true causes of phenomena and the 'form' of things could be inductively established. The term 'form' is to be understood here in the metaphysical sense of a general statement, which accounts for the nature ('ousia') of a phenomenon, such as the form of heat, the form of quicksilver, etc. It is more than a physical law which describes cause-effect relationships between phenomena. He stated that 'the true business of philosophy must be ... to apply the understanding ... to a fresh examination of particulars' [7]. Bacon suggested 27 applications (crucial experiments) for the inductive method that he had outlined.

In the ‘table of presence’ one lists all instances where both a given phenomenon and its observable characteristics are present. Bacon provided a detailed application to the phenomenon of heat. He cited the following affirmative instances in the table of presences: 'the rays of the sun particularly in the summer and at noon', 'sparks arising from the violent percussion of flint on steel', the effect of friction, the effect of physical exercise [7], etc. All these instances are accompanied by the sensation of heat. In order to be true, a proposition (in this case about heat) must necessarily satisfy all items in the table of presence. In other words, the table of presence allows verifying a proposition. For example, the proposition that 'heat is a form of (visible) light' is not verified by the last entry in the table which relates to the sensation of body heat. Hence, the proposition must be discarded, as there is an instance of heat without (visible) light. Heat is not a necessary condition for light.

The ‘table of absence’ provides instances where an observable characteristic of a given phenomenon is present, but where its sensation is absent. In the case of heat, Bacon stated in his table of absence that 'the rays of the moon, stars and comets are not found to be warm at the touch, even when passed through the strongest burning glass' [7]. In order to be valid, a proposition (in this case about heat) must also not satisfy any of the items in the table of absence. This table is most important in the inductive method, as it has the potential to falsify a proposition. The proposition that '(perceived) heat is a form of light' must be rejected, as there is an instance of light without heat. Heat is also not a sufficient condition for light.

The ‘table of degrees’ provides instances in which a greater or lesser degree of a property are accompanied buy a greater or lesser amount of another characteristic. For example in the case of heat he observed that 'animals become more warm by motion and exercise, wine and feasting, venery, burning fevers and grief' [7].

After having listed all the affirmative, negative and comparative accompaniments of a property, Bacon embarked on the difficult task of formulating a 'first vintage' of the 'form' or underlying nature of the phenomenon. This was done by careful examination of the tables of presence and absence. Only that which is shared by all elements in the table of presence and that which is lacking in the table of absence survives scrutiny. What remains is both a necessary and sufficient condition for the phenomenon to take place. (A statement is necessary when it satisfies all items in the table of presence, and sufficient when it satisfies none of the items in the table of absence.) In his study of heat, which we discussed above, Bacon eliminated the possibility that it was a purely terrestrial phenomenon, as the rays from the sun are mentioned in the table of absence. He also ruled out light as the cause of heat on the basis of the table of absence, as the light emanating from the moon could not be felt [2]. He finally concluded that the 'form' of heat is 'an expansive motion restrained, and striving to exert itself in the smaller particles' [7].

The 'form' of heat, as a kind of molecular motion, discovered by Francis Bacon is astonishing when considering the prevailing theories which assumed the existence of a caloric substance. His idea was confirmed in the 19th century by the kinetic theory of gases of Ludwig Boltzmann [9]. Bacon is also credited for having discovered the influence of the moon on the tides and for the geological basis of continental drift [2].

3.2.5. Latent structures

The impact of Bacon's scientific methodology and his plan for organized science found little if any resonance in his own time. His ideas were taken up again, however, two centuries years later by John Stuart Mill [10]. The latter reintroduced Bacon's three tables under the denominations of the 'Joint Method of Agreement and Disagreement' and the 'Method of concomitant Variation'. According to Quinton [2], however, there is a fundamental distinction between the positions of Bacon and Mill, which is very relevant to our later discussion of factor analysis of tabulated data.

Mill and empiricist philosophers, such as Hume [11], aimed to discover observable antecedent causes of the phenomenon under investigation. Bacon's objective was to discover basic explanatory latent factors, which are inaccessible to direct observation. Hence, the form of a thing, according to Bacon, is to be understood as a latent structural property of the matter of which it is composed. It is not some other thing that can be observed as an antecedent cause of the original thing. Bacon also conceived of the form as a latent property which resides within matter (as meant by Aristotle) and less as an abstract quality in a separate world of pure forms or ideas (as proposed by Plato). In later chapters on factor analysis, we will find a close analogy between the forms derived from the tables of presence and absence, on the one hand, and the latent variables (factors) that are extracted mathematically from tabulated data, on the other hand. This is all the more astonishing, considering that Bacon was notoriously unaware of mathematical formalism and produced little if any quantitative data in support of his method.

Francis Bacon died at the age of 66, probably from pneumonia, in the winter of 1626, after the performance of an experiment on the effect of cold in the preservation of meat.

Notes on Bacon

[1] Deduction and induction can be regarded as complementary although radically different branches of what is called the arch of knowledge.
Induction represents the ascending branch, starting from experimental data and ending with hypotheses (from particulars to the general). Deduction constitutes the descending branch which originates from general laws the consequences of which can be verified by means of experiments (from the general to particulars). The arch concept transpires throughout the history of science and is discussed in great detail in:
David Oldroyd, The Arch of Knowledge. An introductory Study of the History of the Philosophy and Methodology of Science. Methuen, New York,1986.

[2] A concise and critical biography has been prepared by:
Anthony Quinton, Francis Bacon. Oxford University Press, Oxford, 1980.

[3] Petrus Ramus (de la Ramée) (1515-1572). French Protestant philosopher and mathematician, who exerted a great influence on the precursors of the Enlightenment. He was a victim of the massacre of the protestant Huguenots during the Bartolomeus night in Paris in 1572.

[5] The center of the utopian state in the 'New Atlantis' was called Salomon's House and was imagined on a forested island in the South Pacific. The highly civilized inhabitants had learned to master science and technology for their own welfare and to the glorification of God. It served as a model for the Royal Society that was founded in England in 1660.
Tore Fraengsmyr (ed.), Solomon's House revisited. The Organization and Institutionalization of Science. Science History (Watson), Canton, MA, 1990.

[6] Against the delicate, disputatious and fantastic learning of his time, Bacon proposed a co-operative and organized activity leading from empirical observations and to the induction of a general laws ('forms'). There is, however, no guarantee that naive induction will produce 'certain' knowledge. One single exception to an inductively derived law will suffice to falsify it. Bacon attempted to guard against this pitfall by means of his table of absence, which requires that there should be no exceptions to the law. The critical inductionism of Karl Popper introduced the concept of falsifiability as a criterion for the scientific elaboration of general statements.
K.R. Popper, Conjectures and Refutations, London, 1963.

[7] Francis Bacon (Lord Verulam), Novum Organum or true Suggestions for the Interpretation of Nature, London, 1620. Reprinted by William Pickering, London, 1844.

[8] Edward de Bono, Francis Bacon. In: The greatest Thinkers. Weidenfeld and Nicholson, London, 1976.

[9] Ludwig Boltzmann (1844-1906) derived the distribution of particles with respect to their energy. From the so-called Maxwell-Boltzmann law, one can derive the macroscopic properties of matter (such as energy, pressure, temperature, density) from the microscopic interactions (elastic collisions) of molecules.

[10] John Stuart Mill (1806-1873), philosopher and economist who received influences from Adam Smith, August Comte and David Ricardo. He is considered as the last of the classical systematic philosophers. In 'A System of Logic' (1843) he described a method by which 'certain' causal relationships could be obtained by means of induction. The elements of his Method bear resemblance with those prescribed by Francis Bacon, although the latter aimed at discovering underlying structure rather than causal laws.

[11] David Hume (1711-1776) is described as a radical empiricist philosopher. In 'A Treatise of human Nature' (1739-40) he proposed that all ideas derive from internal or external experience. In particular, cause-effect relationships arise from our habitual experience that one thing precedes another.

Biographical Notes on Bacon

1561 Born in London, son of Sir Nicholas Bacon, the Lord Keeper.

1573 Entered Trinity College at Cambridge, where he stayed for two years.

1582 Became a barrister and entered the House of Commons. Start of relentless requests for favours and preferment.

1597 Commencement of the literary 'Essays' which were continued until the end of his life.

1605 'The Advancement of Learning', a treatise on the philosophy of science, with a classification of the different forms of knowledge.

1606 Marriage that remained childless.

1607 Appointed Solicitor-General under the reign of King James I

1609 'De Sapientia Veterum', a rational interpretation of ancient myths and fables. Translated into English as 'The Wisdom of the Ancients' in 1619.

1610 'New Atlantis', on the social nature of science, published posthumously.

1612 Obtained the position of Attorney-General, a fulfilment of his life-long aspiration. Supporter of the royal prerogatives against ancient rights and customs.

1617 Obtained his father's former position of Lord Keeper.

1618 Appointed Lord Chancellor, the highest legal position, and made Lord Verulam.

1620 'The great Instauration' and 'Novum Organum', his most important contribution to the philosophy and methodology of science.

1621 Charged with bribery and imprisoned (for a few days) in the Tower of London, which put an end to his professional and political career.

1621-22 'History of Henry VII'

1622 'Historia Ventorum' followed by 'Historiae Vitae et Mortis', compilations of natural history and raw material to be worked out by the method proposed in the Novum Organum.

1623 'De Dignitate et Augmentis Scientiarum', an enlargement of 'The Advancement of Learning'.

1626 Died at Highgate at the age of 66.

3. René Descartes and Coordinate Geometry.

3.3.1. Life and work of Descartes

René Descartes was born in 1596 at La Haye in the region of Tours, in a family of lower nobility, his father being a councillor of the parliament of Bretagne. In later life Descartes referred to himself occasionally as Seigneur du Perron, after the name of his family estate (Fig 3.3.1). He is described as a precocious and inquisitive child who used to nag his father with unremitting questions about 'the reasons of things and their causes', which earned him the name of 'little philosopher' [1]. At the age of eight he was sent to the prestigious and elitist Royal College of La Flèche, which was founded by King Henry IV and run by the Jesuit fathers. Because of delicate health he enjoyed certain indulgent exceptions to the college discipline, by which he was exempt from assisting to morning courses and allowed to remain in bed until noon. Descartes kept to the habit of working from his bed throughout his entire life, even when his health problems had disappeared [2]. He stayed ten years at La Flèche, where he became acquainted with the scholastic philosophy of Aristotle, the metaphysics of Thomas Aquinas and the new advances in mathematics and astronomy.

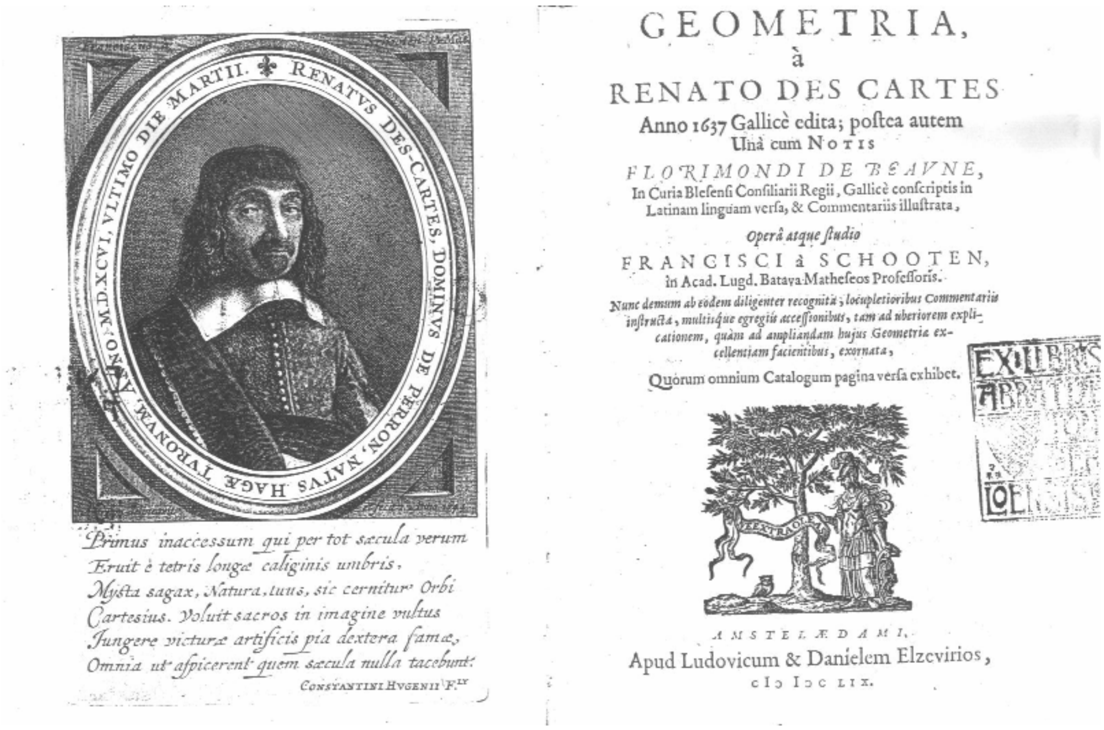


Figure 3.3.1. Frontispiece of the 'Geometria', a Latin translation of the original French version by Florimond de Beaune which was published by Elsevier in Amsterdam in 1659. The engraving was performed earlier by Frans van Schooten, professor of mathematics at Leyden and a close friend of Descartes. The latter had remarked that the beard and the apparel did not resemble to the original.

According to his own report, Descartes felt utterly disappointed and confused by the scholastic teachings that he received, and only found consolation in mathematical courses 'because of the self-evidence of its reasonings'. Instead of considering himself as a learned scholar at the end of his studies at La Flèche, he concluded that he was fully ignorant. He decided to do away with all acquired stuffy knowledge (tabula rasa) and trust only what appeared to his mind as clear and distinct. The determination to find his own way, independently from others and relying only on his own material and mental resources, seems to have been the prime determinant of Descartes' character. The only inescapable certainty was his own doubting, hence his famous maxim 'cogito, sum' (I think, thus I am).

After two years of preparation, Descartes obtained a license degree in civil and canonical law at the University of Poitiers, where he also followed introductory courses in medicine. But, much to the regret of his father, he refused to take up a career as a lawyer, whereupon his father reflected that 'he was only fit to be bound in calf-skin'. Instead, he spent two years in Paris, where he took part in the turbulent life of the literary and scientific avant-garde of his time, which gathered, as they still do today, in worldly salons and taverns. An important event of that period was his encounter with Marin Mersenne, Father of the order of the Minimes, who introduced him into influential circles and who became his permanent contact with the world of scholars.

In 1618 Descartes decided to leave the busy life of Paris for a place where he could reflect on the future direction of his life. In those times, it seems, the profession of a soldier might have attracted those who wished to withdraw from the trivialities of civilian life. Indeed, wars and sieges protracted over several decades. The eighty years war, which opposed the catholic South against the Protestant North, had started in 1568 and still had to be carried on for thirty more years. From this point of view, it is not surprising, perhaps, that Descartes joined the garrison of Prince Maurits of Nassau in Breda, in the south of the United Provinces, as a volunteer-officer. The Protestant Prince, who was in favour of rational conduct of warfare, had already enlisted the services of Simon Stevin for the design of fortifications and army logistics. Descartes refused any form of remuneration, accepting only a symbolic doubloon, and from his own means provided for arms and the services of a mercenary. In return, he obtained the liberty to quit the army at any time and on his own accord.

A decisive event in the life of Descartes was his encounter in 1618 at Breda with Isaac Beeckman (1588-1637), a renowned Dutch physicist and mathematician. The latter aroused and stimulated his interest in problems of physics and mathematics, such as the distance travelled by a free-falling body during a given lapse of time, the pressure inside a liquid at various depths, etc. Descartes applied himself also to find solutions to equations of the third degree, independently of previous work by Vieta, Tartaglia and Cardano [3]. In the same year Descartes composed a 'Compendium Musicae' which he affectedly dedicated to Beeckman. Unfortunately, this work is to become a source of dissent which ended their friendship a few years later, when Beeckman claimed some of the ideas proposed in it to be his own.

In 1619, Descartes terminated his services to the Prince of Nassau and departed for Germany where he joined the army of Duke Maximilian of Bavaria, a supporter of the Catholic Emperor Ferdinand, at the outbreak of the thirty years war against the Protestant insurrection in Bohemia. During the severe winter of that year hostilities ceased and he found himself isolated in a comfortable Bavarian quarter, left entirely to reflection and meditation. In the night of the 10th of November Descartes fell subject to a mystical experience in the form of three rapidly succeeding dreams, which he has recounted accurately [4]. In the first dream he marched with extreme difficulty against a violent wind and encounters a person which appeared familiar to him. The latter seemed unaffected by the tempest and offered an exotic melon. In the second dream he experienced a tremendous explosion as from a lightning that struck inside his room. The last dream was more comforting and confronted him with a classical book of poems and a verse which reads: 'Quod vitae sequabor iter' (What course in life will I pursue). An unknown person appeared and pointed to another verse saying 'Est et Non' (Yes and No). The dreams marked a decisive turning point in the life of Descartes. According to his own interpretation, the tempest signified a force which pushed him in a direction which he had not yet chosen by his own accord. The melon symbolized the charms of solitude, and the lightning was a revelation of the spirit of truth which had descended upon him. The 'Est et Non' pointed to the distinction between truth and falsity which he had to discern in all human knowledge and understanding [1].

The following year, in 1620, Descartes bade farewell to the arms and engaged in extensive, but scarcely documented, travel throughout Europe during which he developed the essential parts of his new philosophy. A first draft of the latter was first published upon his return to France in 1628 as 'Regulae ad Directionem Ingenii' (Rules for the Guidance of Reason). The starting point of his method was the intuition of ideas that are clear and distinct to the mind and from which necessary consequences can be deduced. These in turn should lead to new intuitions, etc. The same year, Descartes left France once again for Holland, but definitively this time.

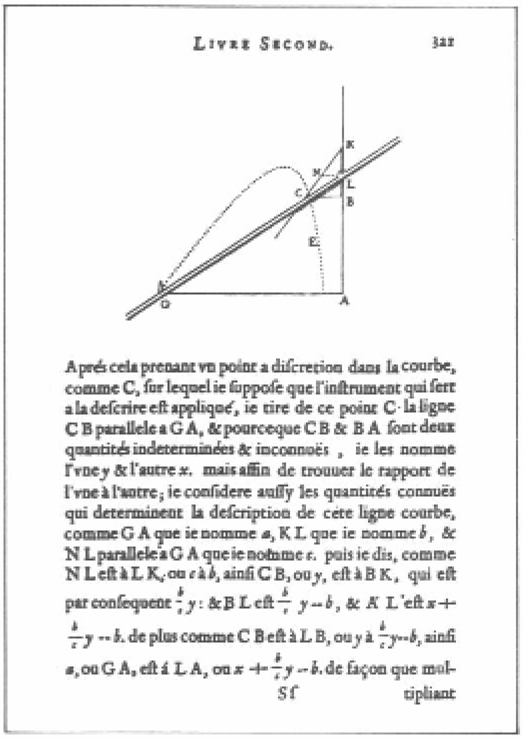
3.3.2. Coordinate geometry

Descartes took various residences in Holland [6] while composing his masterpiece, the 'Discours de la Méthode pour bien conduire sa Raison et pour découvrir la Vérité dans les Sciences' (Treatise of a Method for the good Conduct of one's Reason and for the Search of Truth in the Sciences) which appeared in 1637 [5]. The work had been ready four years earlier, but its publication was delayed by the news of the condemnation of Galilei in Rome. Descartes feared that his work might also be interpreted as an attack on the doctrines of the Catholic Church. For this reason he seeked and obtained approbation of the ecclesiastical authorities in France. Like Oresme, some 300 years earlier, Descartes chose to publish his philosophical synthesis in the language of the people rather than in that of the scholars, in order to reach the largest possible readership [7].

The 'Discours de la Méthode' itself is a rather brief essay of the new philosophy, which is based on four basic rules. The first rule has already been mentioned before. It requires to refuse any knowledge that is not perceived clearly and distinctly by the mind, and to accept only that which cannot be doubted about. The second one demands to divide a problem in as many small parts as is required in order for them to be amenable to a solution. (This could be called a top-down approach.) The third rule advises to proceed in an orderly fashion, by directing thoughts to the more simple objects and progressing step-by-step towards the more complex ones. (This approach could be termed as bottom-up.) Descartes emphasized that one should always assume that there is order in successive steps even if that order does not appear to be natural. Finally, the last rule prescribes to perform a general review and complete enumeration of all possible cases and contingencies.

In Descartes' time mathematics still largely depended upon geometrical demonstrations according to the ancient Greek tradition ('via antiqua'), while the algebraic method ('via moderna') was still in the process of development. Descartes contributed to our modern algebraic notation, notably in the definition of powers of an unknown quantity such as x^2, x^3, etc. which were previously written as xx, xxx, etc. One of the great mathematical achievements of Descartes was the synthesis of geometry and algebra, which is announced in the 'Discours de la Méthode'. He found no reason why there should be different types of mathematics, and conceived of a universal mathematics in which the shortcomings of geometry and algebra would be compensated by each other.

The major part of the 'Discours de la Méthode' is taken up by three rather extensive appendices which contain applications of the method, i.e. 'Dioptrique' (optics), 'Météores' (phenomena of the sky) and 'Géométrie'. In the latter, Descartes makes use of his mathematical invention which is to express geometrical constructs, such as line segments, by means of algebraic symbols which represent coordinates that can be subjected to arithmetic operations. We refer to this method as coordinate geometry, although others prefer the more general term of analytical geometry. In the first part of the 'Géométrie' Descartes explained how the arithmetic operations of multiplication, division and extraction of square roots can be performed by means of compass and ruler. The second part is the most relevant to our discussion as it deals with algebraic equations that represent the locus of a geometric construction, particularly equations of the second degree (conic sections). Finally, the last part discusses various problems concerning solids.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Aprés cela prenant vn point a discretion dans la courbe, comme C, sur lequel ie suppose que l'instrument qui sert a la descrire est appliqué, ie tire de ce point C la ligne C B parallele a G A, & pourceque C B & B A sont deux quantités indeterminées & inconnuës, ie les nomme l'vne y & l'autre x. mais affin de trouuer le rapport de l'vne à l'autre; ie considere aussy les quantités connuës qui determinent la description de cete ligne courbe, comme G A que ie nomme a, K L que ie nomme b, & N L parallele a G A que ie nomme c. puis ie dis, comme N L est à L K, ou c à b, ainsi C B, ou y, est à B K, qui est par consequent $\frac{b}{c}y$: & B L est $\frac{b}{c}y - b$, & A L est $x + \frac{b}{c}y - b$. de plus comme C B est à L B, ou y à $\frac{b}{c}y - b$, ainsi a, ou G A, est à L A, ou $x + \frac{b}{c}y - b$. de façon que mul-

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Figure 3.3.2. Reproduction of a page from the original 'Géométrie', which deals with the problem of the hyperbola that is generated by two intersecting mobile lines GL and KN. The variable line segments AB and CB were replaced by the algebraic unknown quantities x and y, respectively. Substitution of these in the geometric relations that derive from the similarities of the triangles NKL, NCB and LCB, LGA leads to a quadratic equation in x and y, which defines a hyperbola in a rectangular system of coordinate axes.

By way of illustration we discuss the application of the method to the construction of a hyperbola, following the original notation of the 'Géométrie' (Fig. 3.3.2). The problem is to determine the curve that results from the intersection of two mobile lines. The first mobile line GL pivots at the fixed point G on the horizontal line GA and slides along the vertical line AL at the variable point L. The second mobile line KN makes a constant angle with the vertical line and slides along this line at the point L. The distance KL is assumed to be constant. Let the variable point of intersection between the two lines be called C. Descartes made the following substitutions for the fixed line segments: GA = a, KL = b, NL =c and for the variable line segments: AB = x, CB = y. From the similarities of the triangles KNL and KCB now follows that:

$$NL/LK = CB/BK \quad \text{or} \quad c/b = y/BK$$

From the similarity of the triangles LCB and LGA follows that:

$$cb/BL = GA/LA \quad \text{or} \quad y/BK - b = a/x + BK - b$$

Elimination of BK from the two relations and rearrangement of terms leads to the quadratic equation:

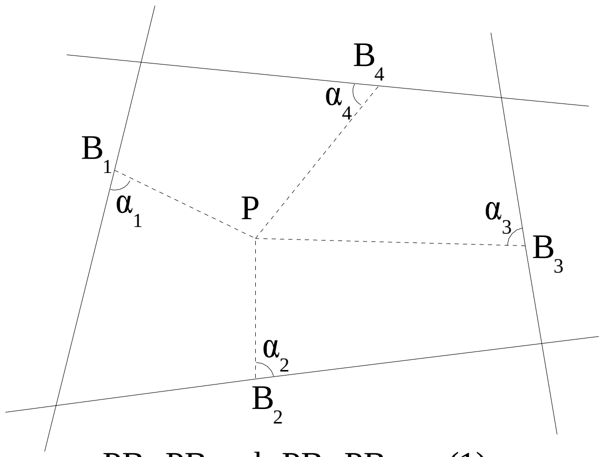
$$y^2 + (c/b)xy - (a + c)\, y + ac = 0$$

This defines a hyperbolic curve in the unknowns x and y with the fixed parameters a, b and c. The daring step of Descartes was to use algebraic symbols, such as x and y, for appropriately chosen variable line segments and then to apply algebraic operations to them. The graphic representation of the relationship between the unknowns x and y aids in the perception of an (a priori) truth which is revealed to the mind by means of deduction. It is distinct from the representation of empirical facts that are obtained by observation and measurement. Descartes advocated the use of graphical representation in order to visualize relationships between several variables in a way which is clear and distinct [8].

Nowadays we refer to the unknowns x and y as the coordinates of a variable point on the curve. The lines that carry the unknown quantities are now referred to as coordinate axes. The fact that the coordinate axes make a right angle in the problem of the hyperbolic curve is coincidental. In a demonstration of Pappus theorem Descartes considers the general case were the carrier lines of the unknowns x and y make an arbitrary angle. The theorem considers four arbitrarily fixed lines a, b, c, d and a variable point P from which four line segments PA, PB, PC, PD are drawn that intersect the lines at a constant angle (Fig. 3.3.3). Descartes proved that a conic section is described by the locus of the variable point P for which the line segments satisfy the relation:

PA/PB = PC/PD

The problem was solved in this case by taking the intersection of any two lines, say a and b, as the origin O of an oblique system of coordinate axes and to replace the variable line segments OA and OB by the unknown algebraic quantities x and y. By substituting the line segments PA, PB, PC, PD by their algebraic equivalents a quadratic equation in x and y is obtained, which proved Pappus theorem that the locus of the variable point P describes a conic section.



$$PB_1 \cdot PB_2 = k \cdot PB_3 \cdot PB_4 \qquad (1)$$

Figure 3.3.3. Application of coordinate geometry to Pappus theorem, which states that the locus of the variable point P is on a conic section given that the lines a, b, c, d are fixed and that the line segments PA, PB, PC, PD are drawn at a constant angle and satisfy the relationship PA/PB = PC/PD. For convenience, the four line segments are drawn at right angles to the fixed lines. The variable line segments OA and OA define the algebraic unknowns x and y which are also the oblique coordinates of the variable point P. Geometrical considerations lead to a quadratic equation in x and y, which proves the theorem. (From Steve Lovell, myweb.tiscali.co.uk/cslphilos/algebra.htm)

3.3.3. The foundation of Cartesianism

The last two great works of Descartes were of a metaphysical and philosophical nature. In the 'Méditations' of 1642 he defines his metaphysical position, particularly the dualism between the soul and the body, which he considers as operating more or less independently of each other [9]. The subtle relationship between the two can only be grasped, according to Descartes, by pure introspection which excludes all forms of meditation and reasoning which appeal to the senses and the imagination. Two years later he published the 'Principia Philosophiae' which emphasizes his mechanistic view of the universe: 'I have described the earth and the whole visible Universe on the model of a machine, without considering anything beyond figures and motion' [10].

From 1639 onwards, a growing opposition towards Cartesianism became manifest in Holland under the instigation of Voetius, a theologian at the University of Utrecht, one of the principal charges being atheism. This may have compelled Descartes to accept in 1649 an invitation to the austere court of Christina X, Queen of Sweden. Four months after his arrival Descartes expired in the glacial royal palace in Stockholm, after a brief spell of sickness, on the 11th of February, 1650. His remains were transferred to Paris and are buried in the Pantheon among those of the greatest that France has produced.

Notes on Descartes

[1] The biographical data on René Descartes have been taken from:
Germaine Lot, René Descartes, Esprit Soleil. Seghers, Paris, 1966.
Encyclopaedia Britannica, Macropaedia, p.598.

[2] Besides literature, philosophy and religion, the College at La Flèche also provided instruction in riding and fencing. Descartes is credited to have produced a manual of fencing (which has been lost) describing a method for getting the better from an opponent in all practical circumstances. There is no doubt that, through sheer will-power, he had returned to good health at the end of his studies. He may have presented, however, a case of 'creative malady' in which a condition of ill-health is simulated in order to free the mind and the body from trivial distractions. A similar situation may have occurred in the later life of Florence Nightingale, whose mysterious neurotic condition prevented her to leave her room from which she directed the health reform in Britain during the second half of the 19th century.

[3] A characteristic of Descartes' way of thinking was that he tried to find several solutions to problems and proofs of theorems, as many as possible and independently of those already proposed by others. He hardly used any books, except for the Elements of Euclid and the Summa of Thomas Aquinas which he borrowed occasionally. This is in stark contrast to the habit of present-day scholars whose collaborative work is organized along the precepts laid out by Francis Bacon in the 'New Atlantis'.

[4] A similar mystical experience befell Blaise Pascal, a contemporary of Descartes, who was suddenly confronted with a bottomless crevice, which deeply affected his spiritual life thereafter. It has been claimed that Descartes was acquainted with the mystical sect of the Rosicruceans, but this insinuation from his opponent Voetius has not been substantiated.

[5] René Descartes, Discours de la Méthode pour bien conduire sa Raison et pour chercher la Vérité dans les Sciences. Jean Maire, Leyden, 1637. A modern edition, based on the original text, appeared by Pierre Cailler, Genève, 1947.

[6] From 1628 to 1649, Descartes frequently moved around in Holland and stayed in Franeker (Friesland), Leyden, Amsterdam, Deventer and Utrecht. He became acquainted, among others, with the poet Constantin Huyghens (father of Christian, the physicist) and the mathematician Frans van Schooten (whose son engraved the figures in the 'Discours de la Méthode). While in Amsterdam, Descartes had a brief relationship with his servant Hélène, who in 1635 gave birth to a girl which was named Francine. Despite the affectionate care of Descartes, his child died a few years later of scarlet fever. The song by Georges Brassens (Les sabots d’Hélène) refers to this episode in the life of Descartes.

[7] Although the use of the algebraic coordinates x and y already appears familiar to the modern reader, the mechanical definition of a curve by means of a moving point is typically in the style used by geometricians in Descartes' time.

[8] Kuehn F.R., Deskartes' Verhaeltnis zur Mathematik, Roesl, Muenchen, 1923.

[9] Descartes proposed that the pineal gland in the brain is the place where the soul enters in contact with the body [1].

[10] William R. Shea, The Magic of Number and Motion. The scientific Career of René Descartes. Watson, 1991. Review by A. Rupert Hall, The power of invention. Nature, 352, 675, 1991.

Biograpical Notes on Descartes

1596 Born at La Haye in Touraine (Bretagne) from a family of lower nobility.

1604 Entered the Royal College of the Jesuit fathers at la Flèche.

1616 Obtained a degree (license) in civil and canonical law at the University of Poitiers. Descartes spent two years in Paris, where he acquainted with Father Marin Mersenne his life-long friend and corresponding contact with French scholars.
'Manuel d'Escrime' (a manual of fencing), which has been lost.

1618 Left Paris and joined the army of Prince Maurits of Nassau in Breda as a volunteer-officer. Encounter with the eminent physicist and mathematician Isaac Beeckman, who became his mentor.
'Compendium Musicae', dedicated by Descartes to Beeckman and cause of their later discord.

1619 Joined the army of Duke Maximilian of Bavaria, at the outbreak of the thirty years war. Solitary period of reflection and meditation in a winter quarter in Germany.

1620 Left the armed services and travelled throughout Europe.

1628 'Regulae at Directionem Ingenii' , a first draft of his philosophy, published in France.
Returned to Holland where he took residence in various locations.

1637 'Discours de la Méthode pour bien conduire sa Raison et pour découvrir la Vérité dans les Sciences', Jean Maire, Leyden. This philosophical essay is followed by three applications of the method: 'Dioptrique, Météores, Géométrie'.

1642 'Méditations', a metaphysical essay which defines Descartes' dualism between soul and body. It provoked an intense correspondence with Princess Elisabeth, daughter of King Friedrich of Bohemia, exiled in Holland since 1620.

1644 'Principia Philosophiae', which defined the mathematical and mechanical foundation of Cartesianism.

1645 'Passions de l'Ame', an essay on psychology and physiology.

1649 Invited to the court of Christina X of Sweden.

1650 Died in Stockholm, possibly of pneumonia, at the age of 54.
His body is transferred to the Pantheon in Paris.